THE SELECTIVITY OF NOTCH FILTERS USING NONUNIFORM RC LINES*

SYNOPSIS--The notch selectivity of a notch filter is defined as the half-power selectivity of a feedback amplifier with an open-loop gain of 40 db and the notch filter in its feedback circuit. Parameters of the exponential, the trigonometric, and the hyperbolic RC lines for different selectivities are given.

One of the important applications of the uniform and nonuniform RC lines is their use in the notch filters.[1] When an external resistance of appropriate value is connected in series (as two-ports) with the RC line (Fig. 1), a real-frequency transmission zero can be obtained. The series combination will then have a notch transmission characteristic.

For almost all applications in which a notch characteristic is desired, the exact shape of the notch is not important. This is true because almost all notch filters have practically the identical variation in the neighborhood of the notch frequency. The only relevant feature in this region is the rate at which the loss decreases as the frequency is moved away from the notch. Unfortunately, no standard definition exists for a quantity that will effectively serve to compare these notch characteristics. This task is made difficult because this rate is usually different on both sides of the notch and different for variously shaped RC lines. The definition of a quantity analogous to the half-power bandwidth of an RLC series circuit is, therefore, not possible.

For our present purposes, we shall nevertheless define a half-power bandwidth for a notch filter as the half-power bandwidth of a feedback amplifier with an open-loop gain of 40 db and the notch filter in its feedback circuit. The choice of the figure 40-db is entirely arbitrary. This definition is illustrated in Fig. 1 where ω_0 is the notch frequency and ω_1 and ω_2 are the half-power frequencies. It is easy to show that these two frequencies correspond to the 47.6456-db-loss points on the notch characteristic. With this definition, we shall define

$$\text{Notch Selectivity} = \frac{\omega_0}{\omega_2 - \omega_1} \tag{1}$$

It can generally be assumed that notch filters with the same notch selectivity will behave very similarly, if not identically, at the vicinity of the notch frequency.

In a previous communication, it was shown that the three general shapes of RC lines can be analyzed in closed form and quantitative studies are possible without resorting to approximation techniques or infinite series.[2] This letter is intended to show that parameters for these lines can be chosen such that they have exactly the same notch selectivity.

The three RC lines and the notation used in this study are:

(1) The exponential line: $r = r_0\,\epsilon^{\alpha x}$, $c = c_0\,\epsilon^{-\alpha x}$, port 1 is at $x = 0$ and port 2 is at $x = \lambda$.

(2) The trigonometric line: $r = r_0 \csc^2 x$, $c = c_0 \sin^2 x$, port 1 is at $x = x_1$ and port 2 is at $x = x_2$.

(3) The hyperbolic line: $r = r_0\,\mathrm{sech}^2 x$, $c = c_0 \cosh^2 x$, port 1 is at $x = x_1$ and port 2 is at $x = x_2$.

With the definition for selectivity given above and the parameter functions given in the references[1,2,3] the notch selectivity for any given line parameters can be found. Several selectivities for the exponential lines have been found and are given in Table 1. It can be noted that the line corresponding to $\alpha\lambda = 0$ is the uniform line. In Table 1, the values for ω_0 are given as multiples of $1/r_0c_0\lambda^2$ and those for R_s are given as multiples of total resistance of the RC line or

$$\int_0^\lambda r_0\epsilon^{\alpha x}dx \tag{2}$$

It can be seen from Table 1 that tapering gererally increases selectivity.

Table 1

Selectivity	Degree of Taper $\alpha\lambda$	Notch Frequency ω_0	Series Resistance R_s
10	-2.9303	12.305	0.0088653
20	-1.6661	11.561	0.021478
30	-0.78374	11.271	0.036887
40	-0.046626	11.187	0.054884
40.672	0	11.187	0.056184
50	0.62610	11.240	0.075430
60	1.2763	11.408	0.098564
70	1.9331	11.688	0.12439
80	2.6230	12.092	0.15308
90	3.3758	12.648	0.18489
100	4.2324	13.405	0.22020
110	5.2575	14.453	0.25960
120	6.5697	15.969	0.30404
130	8.4276	18.346	0.35527

There are multitudes of parameters for the trigonometric and hyperbolic lines that correspond to a given selectivity. These parameters are best presented as contour maps. These contours are shown in Fig. 2. In these graphs, points on contours of the same value all have the same selectivity. It is understood, of course, that the values of ω_0 and R_s are usually different.

The feedback circuit of Fig. 1 is assumed to draw no input current. Hence, the notch selectivity computed this way are the selectivities of the notch filters when the output is open circuited. In a future communication, the effect of load resistance to the notch selectivity will be studied for these three shaped RC lines.

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Footnote

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References

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2 SU, K. L.: 'Hyperbolic RC transmission line,' Electronics Letters, 1965, 1, p. 59.

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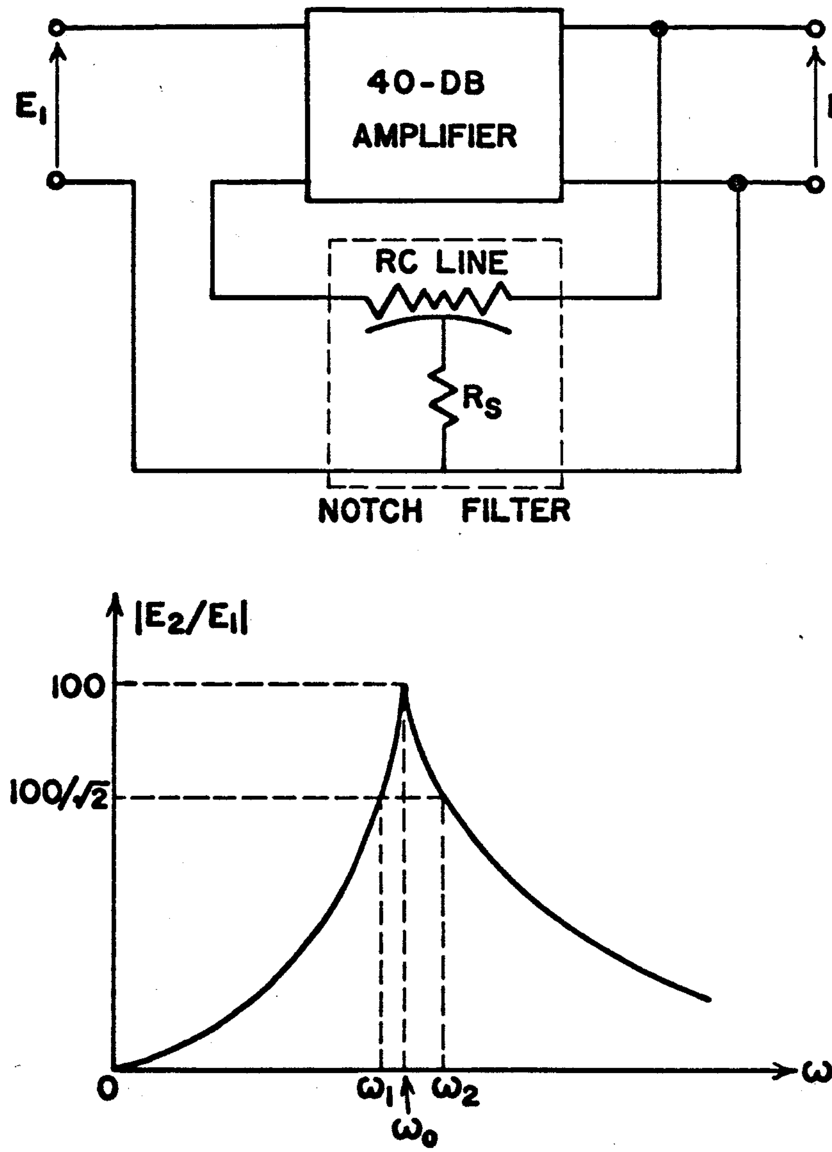
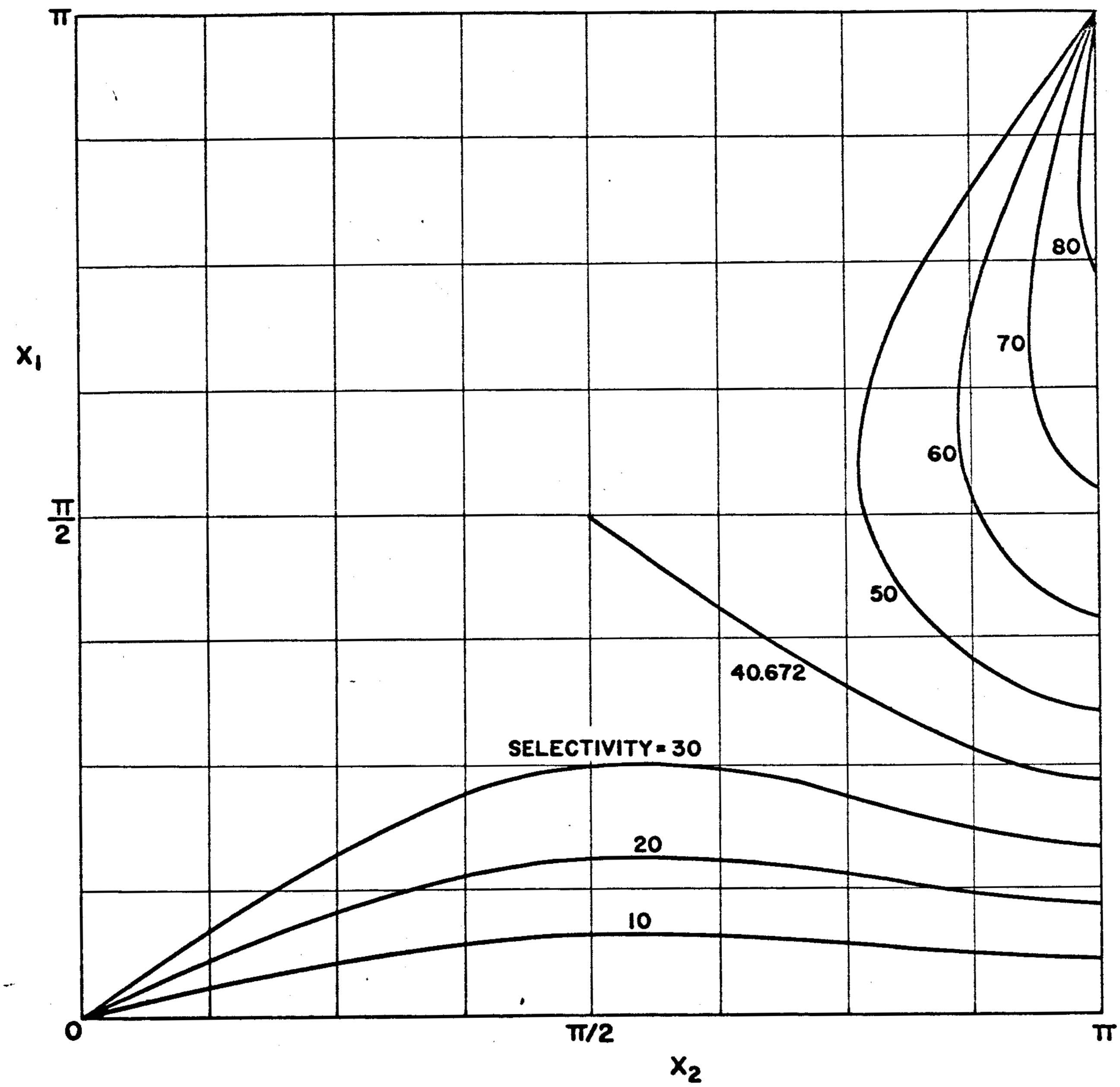
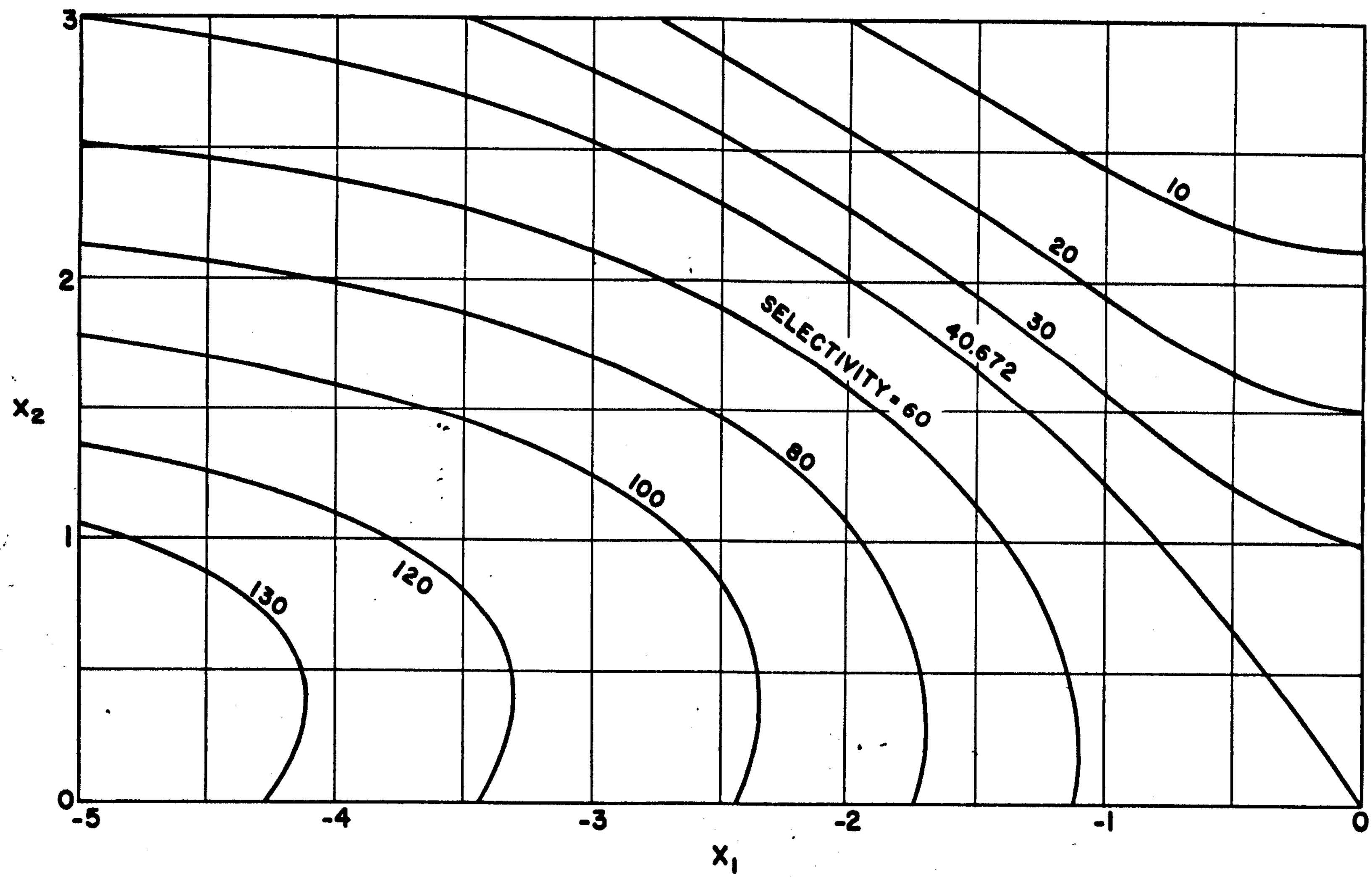


Fig. 1 Amplifier circuit and typical characteristic used to define the notch selectivity.



(a)

Fig. 2 Contour maps of nonuniform RC line parameters with constant notch selectivity. (a) Trigonometric line.



(b)

Fig. 2 Contour maps of nonuniform RC line parameters with constant notch selectivity. (b) Hyperbolic line.